Electric Flux& Gauss Law

Electric Flux

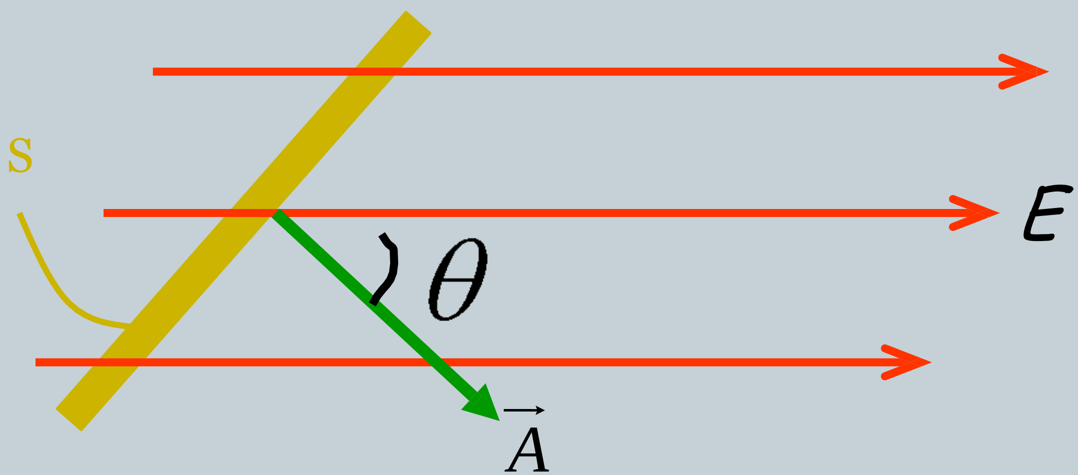
Electric flux is the measure of the ―number of field lines passing through a surface S‖

For uniform : $\vec{E}$

Define: Electric Flux

$$\Phi_E = \mathbf{E} \bullet \mathbf{A}$$

Units: N•m²/C



A is the surface area perpendicular to $\vec{E}$, so Φ=EAcos(θ)

Notes:

1) Φ_E is a scalar called electric flux

2) Units: N•m²/C

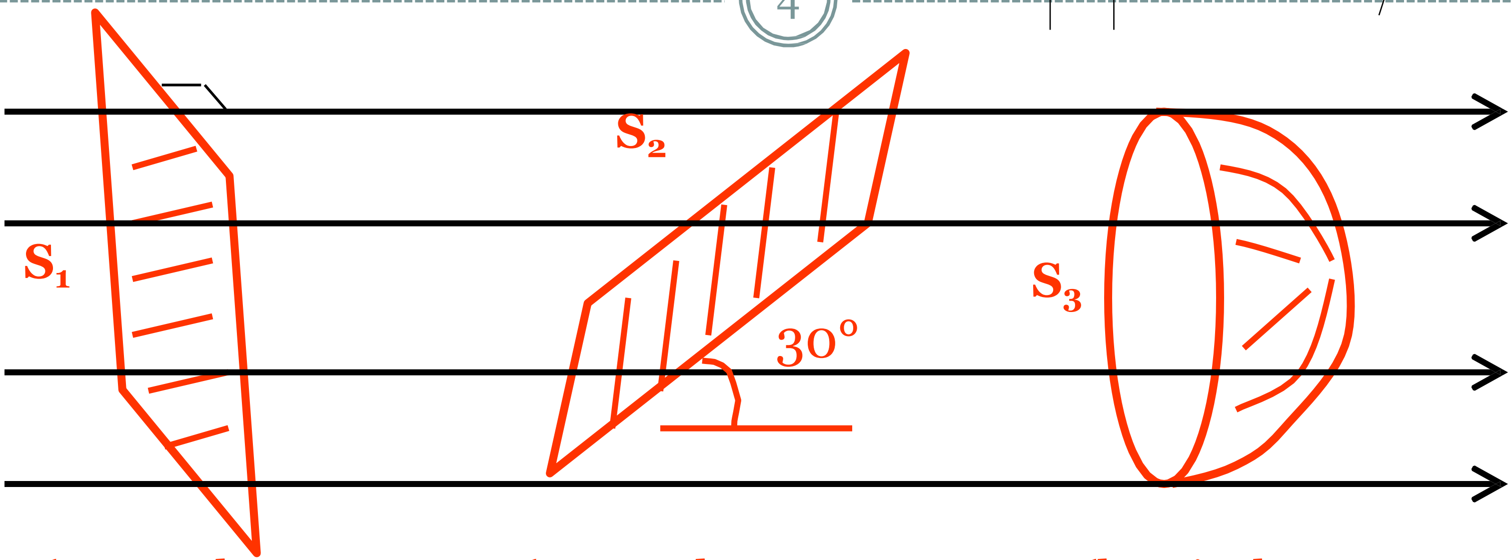
3) Φ_E represents the "number of field lines through surface S"

4) For a closed surface, the area vector points in the outward direction.

5) Flux is zero for a surface parallel to the field (normal is at 90^{o} to $\boldsymbol{E}$)

Example:

$$\left|\vec{E}\right| = 1000\,N/C$$



(rectangle, 1m x 2m) (rectangle, 1m x 2m) (hemisphere, radius 1m)

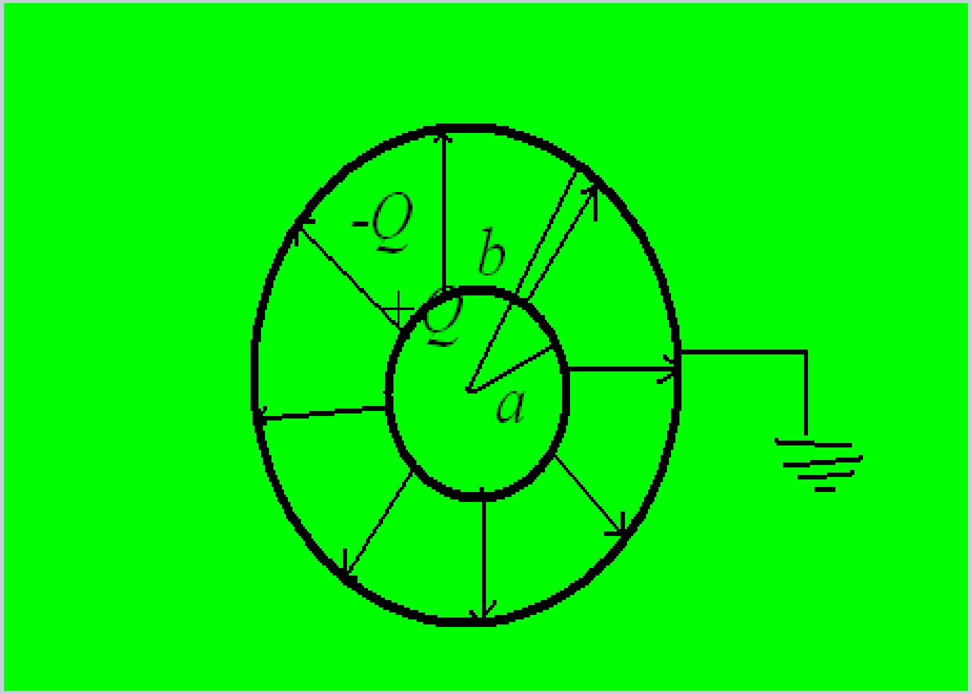
Find: flux Φ_E through S_1, S_2, S_3.

➔Faraday's Experiment:

- From this experiment

  $\psi = Q$------(1)

Now we can find surface charge or flux density



D|r=a =

---for inner sphere.

D|r=b = $\frac{Q}{4\pi b^2} a_r$

---for outer sphere

- D= $\frac{Q}{4\pi r^2} a_r$ --for a<r<b;

➔now, the electric field at any point in free space

E= $\frac{Q}{4\pi\varepsilon_0 r^2} a_r$

➔hence for free space

D=ϵo E

➔Gauss's Law

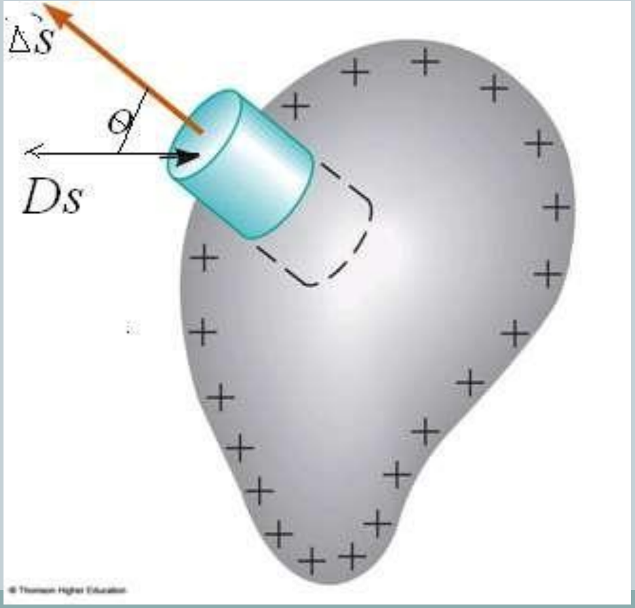
- The Faraday's experiment leads to generalized statement known as Gauss Law

" The Electric flux passing through any closed surface (known as Gaussian surface) is equal to total charge enclosed by the surface."

Mathematically:

$\Delta\psi$*=flux crossing* ΔS

$= D_s \, \Delta S \cos\theta = \mathbf{D_s} \cdot \mathbf{\Delta S}$



- ➔the total flux passing through surface:

$$\Psi = \int d\psi = \oint Ds.ds$$

➔the closed surface is known as Gaussian surface

Ψ=charge enclosed =Q

- ➔ now we know that

$$D = \varepsilon_0 E$$

- ➔ *"The total electric flux through any closed surface equals the net charge inside that surface divided by* ε_o*"*

$$\oint_s E.ds = \frac{q}{\varepsilon_0}$$

- ➔This relates an electric field to the charge distribution that creates it.

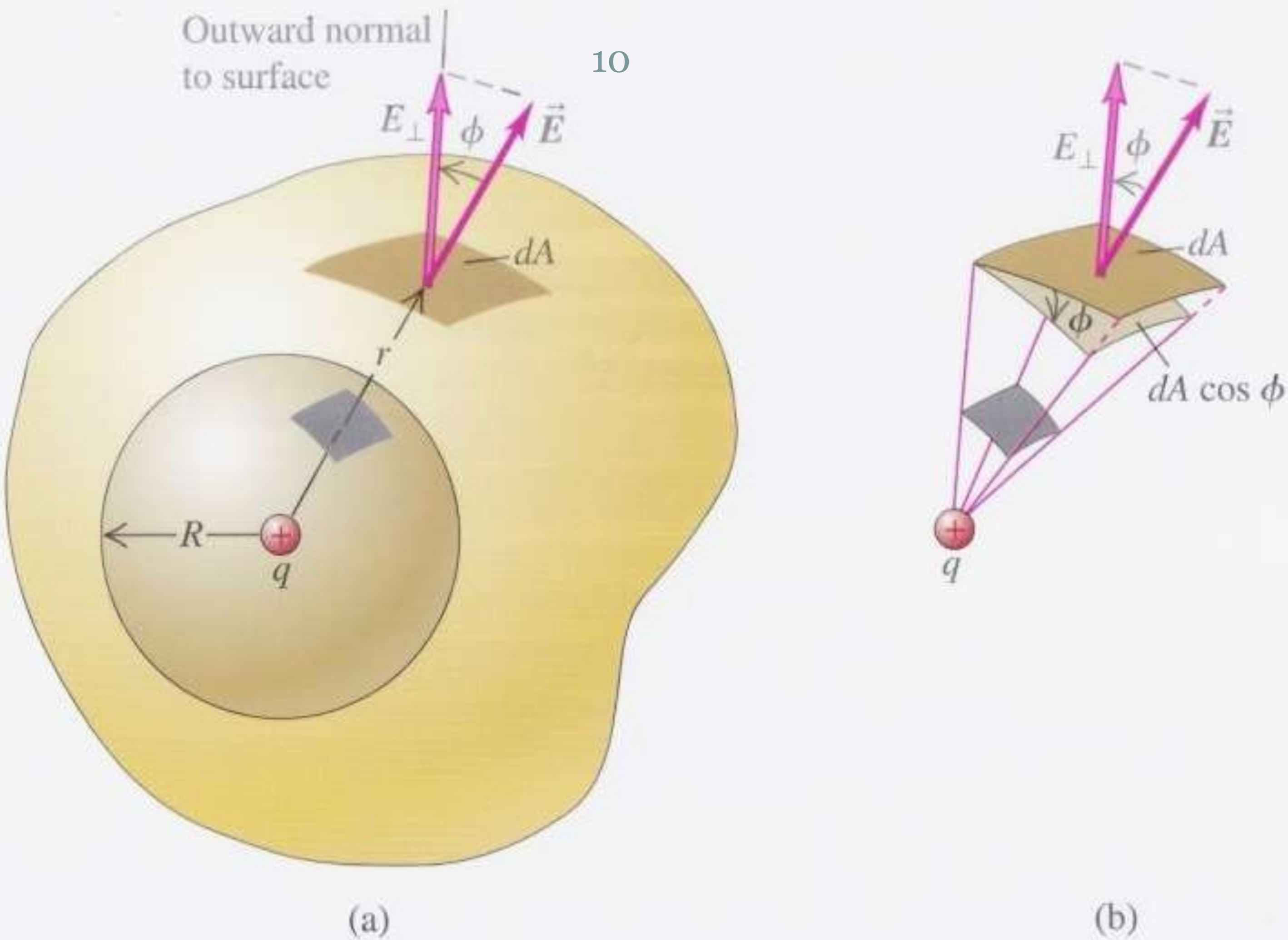


Figure 22.13 Calculating electric flux through an arbitrary surface

➔*Application of Gauss's Law:*

- ➔ By using gauss law we can determine Electric field or Charge density.
- ➔ But the solution is easy if we are able to choose a close surface, which satisfies two conditions :

1: *Ds is everywhere either normal or tangential to the closed surface, so that Ds.ds become either Dsds or Zero.*

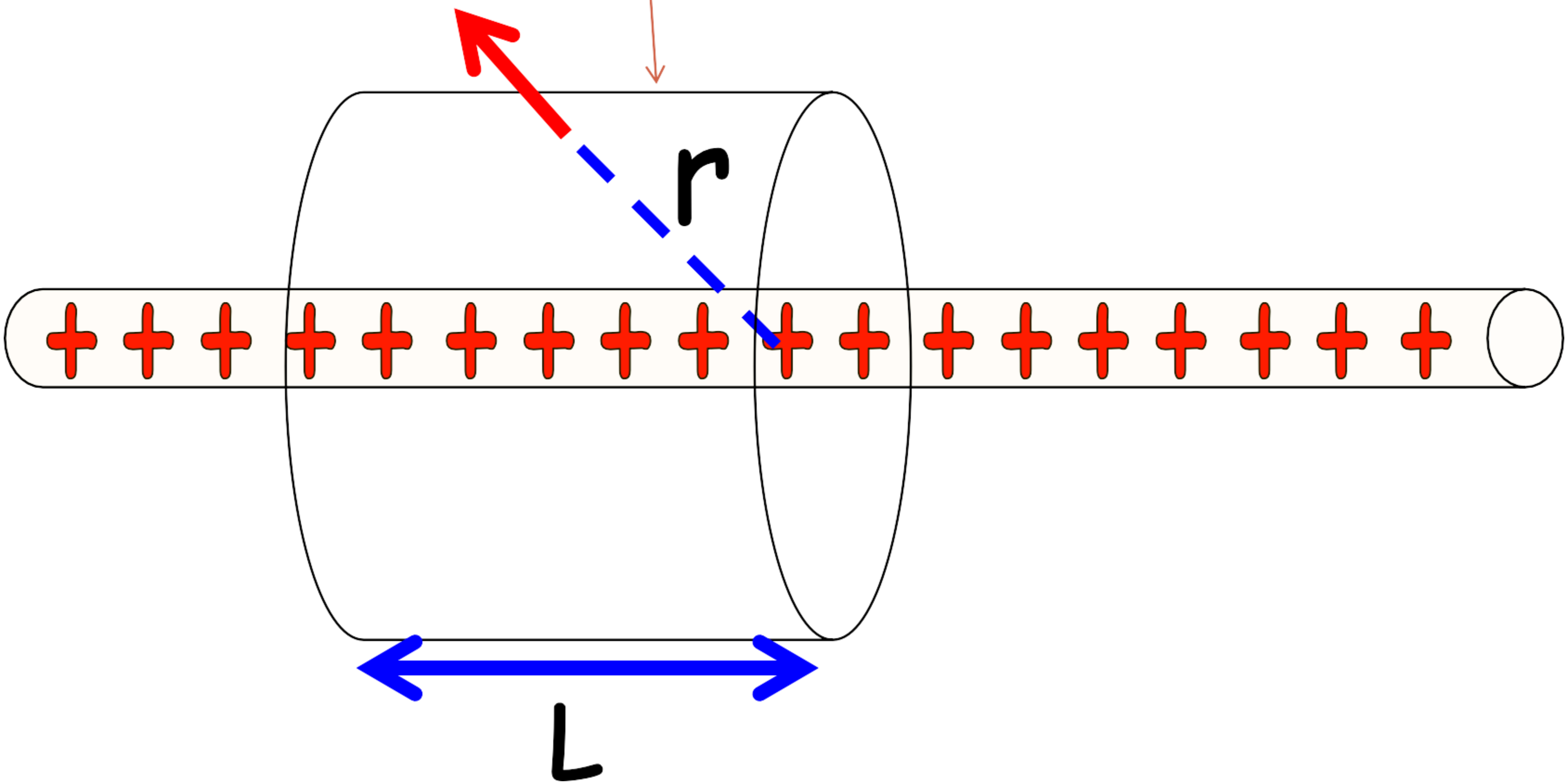
2: *One portion of the closed surface for which Ds.ds is not zero, Ds=constant*

- ➔ Note:

  " Gauss's Law depends upon symmetry, so that if we can not show symmetry exist then we can not use Gauss law"

➔*For Example : Electric field due to line charge distribution:*

- Let a cylindrical closed surface around the charged line having radius 'r'
- side



- Now apply gauss law

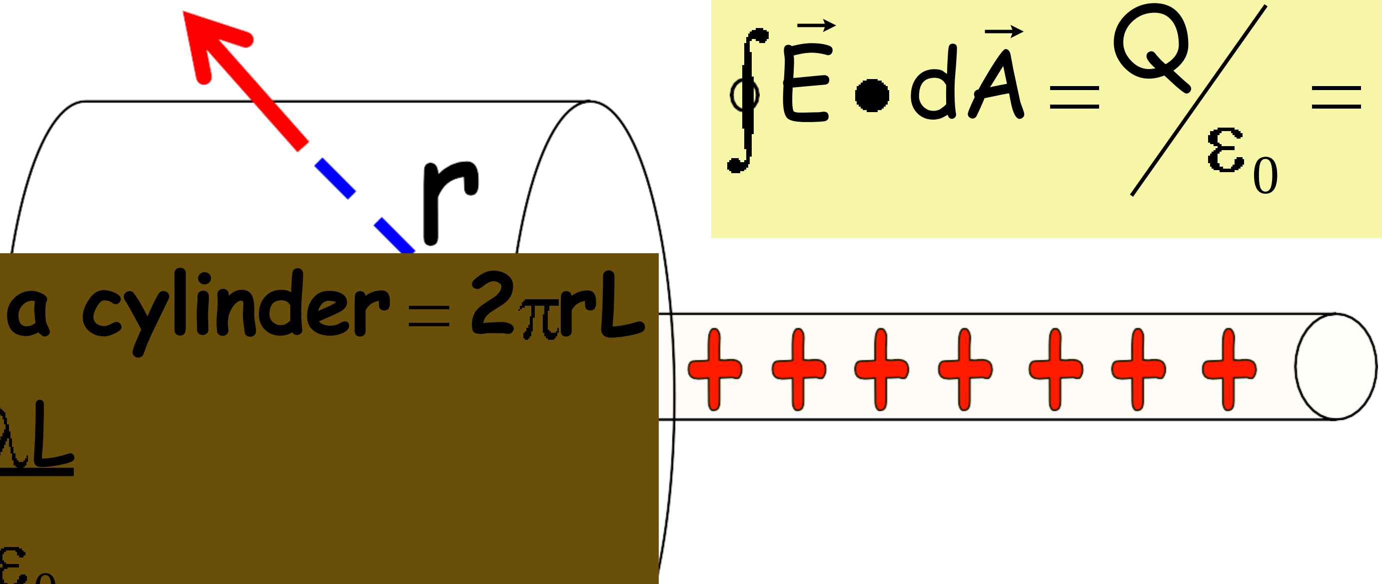
$$Q = \oint_{cyl} Ds.ds = Ds \int_{side} ds + 0.\int_{top} ds + 0.\int_{bottom} ds$$

$$= Ds.2\pi rL$$

$$\Rightarrow Ds = \frac{Q}{2\pi rL}$$

$$\Rightarrow E = \frac{Ds}{\varepsilon_0} = \frac{Q}{2\pi r\varepsilon_0 L} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Exercise 4



Gauss

$$\oint \vec{E} \bullet d\vec{A} = Q/\varepsilon_0 = \frac{\lambda L}{\varepsilon_0}$$

$$\text{Area of a cylinder} = 2\pi r L$$

$$E 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi r \varepsilon_0}$$

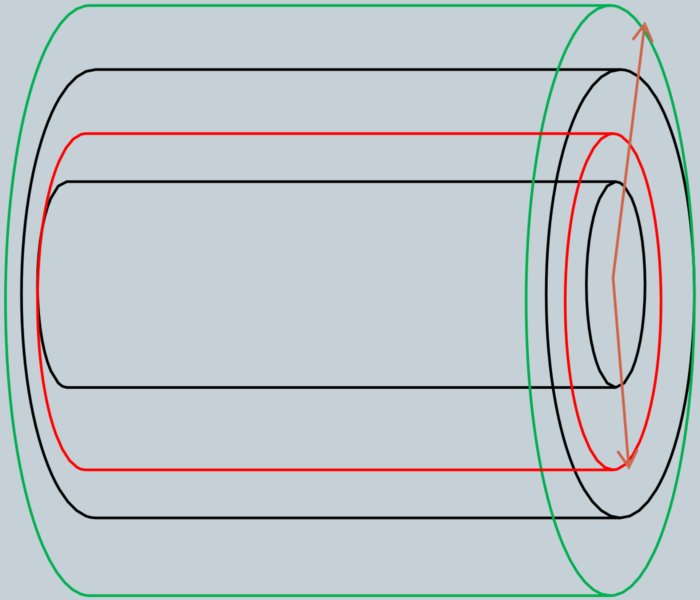
➔Electric field due to Co-axial cable

- Let us consider a coaxial cable having inner radius a and outer radius b. and the outer surface of inner cylinder is *Ps* .
- Now we are interested to find field inside and out side..
- ➔Green surface:

  r>b

➔Red Surface:

  a<r<b



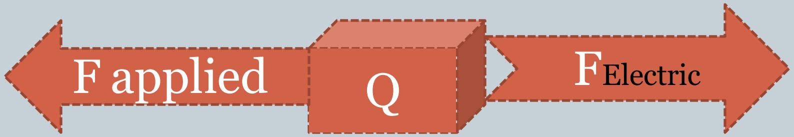
Energy and Potential

➔Energy Expended in Moving A Point Charge In An Electric Field

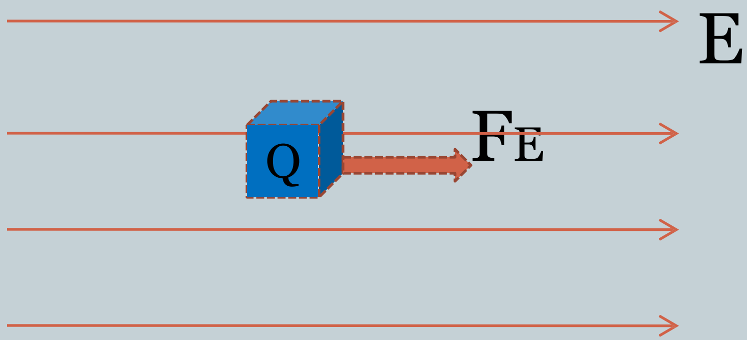
- The Electric field intensity defined as---

" The force on a unit test charge at that point at which we wish to find the value of this vector field"

Now, if we attempt the test charge against the Electric field, we have to exert a force equal and opposite to that exerted by field. And this requires us to expend energy or do work.



- ➔ Now , suppose we wish to move a charge Q a distance *dL* in Electric field E.



$$\vec{F} = QE\hat{a}_l$$

➔ a_l = unit vector in dL direction

- The applied force:

$$F_{applied} = -Q E . \hat{a}_l$$

- The work done

$$dW=-QE.dL$$

- Net work done:

$$W = -Q\int_{initial}^{final} E.dL$$

➔How to perform line integral:

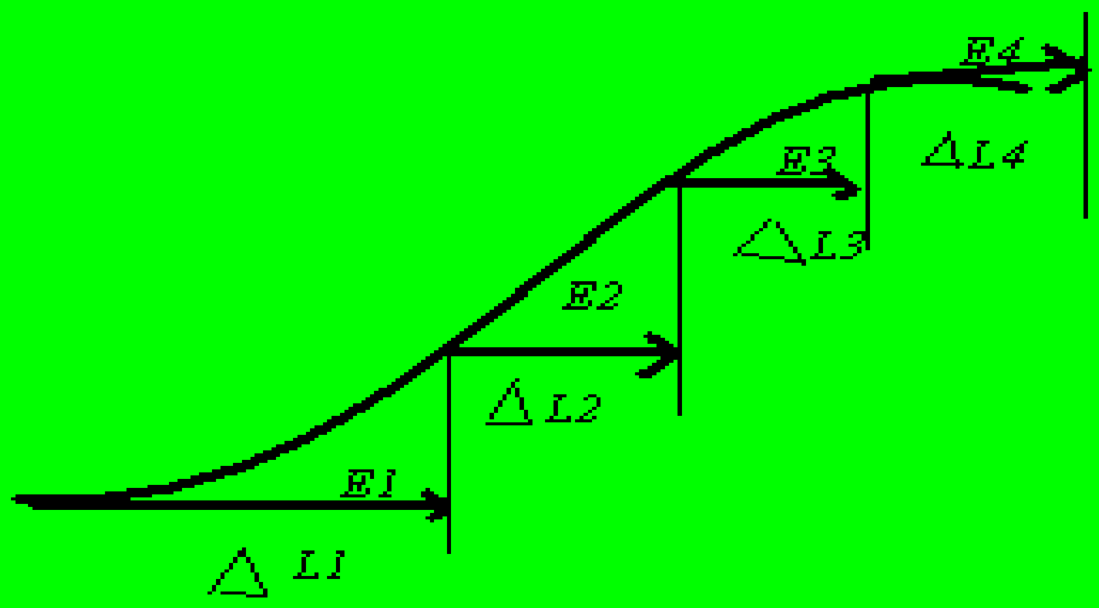
--To perform line integral choose a path , then break it up into large number of very small segments,

-- multiply the component of the field along each segment by the length of the segments

--and add the results for all the segments.

For example :

“Let an uniform Electric field for the simplicity and break the path into segments as in figure”


E1
E2
E3
E4
△ L1
△ L2
△L3
△L4

- Now for $w = -Q \int_{initial}^{final} E \,.dL$

Apply the method:

$$w = -Q(E_1\Delta L_1 + E_2\Delta L_2 + E_3\Delta L_3 + E_4\Delta L_4)$$

For Uniform field

E1=E2=E3=E4=E

➔ $$W = -QE(\Delta L_1 + \Delta L_2 + \Delta L_3 + \Delta L_4)$$

$$\Rightarrow W = -QE.L_{BA}$$

- Now if there are infinite number of elements the sum can be converted as integral:

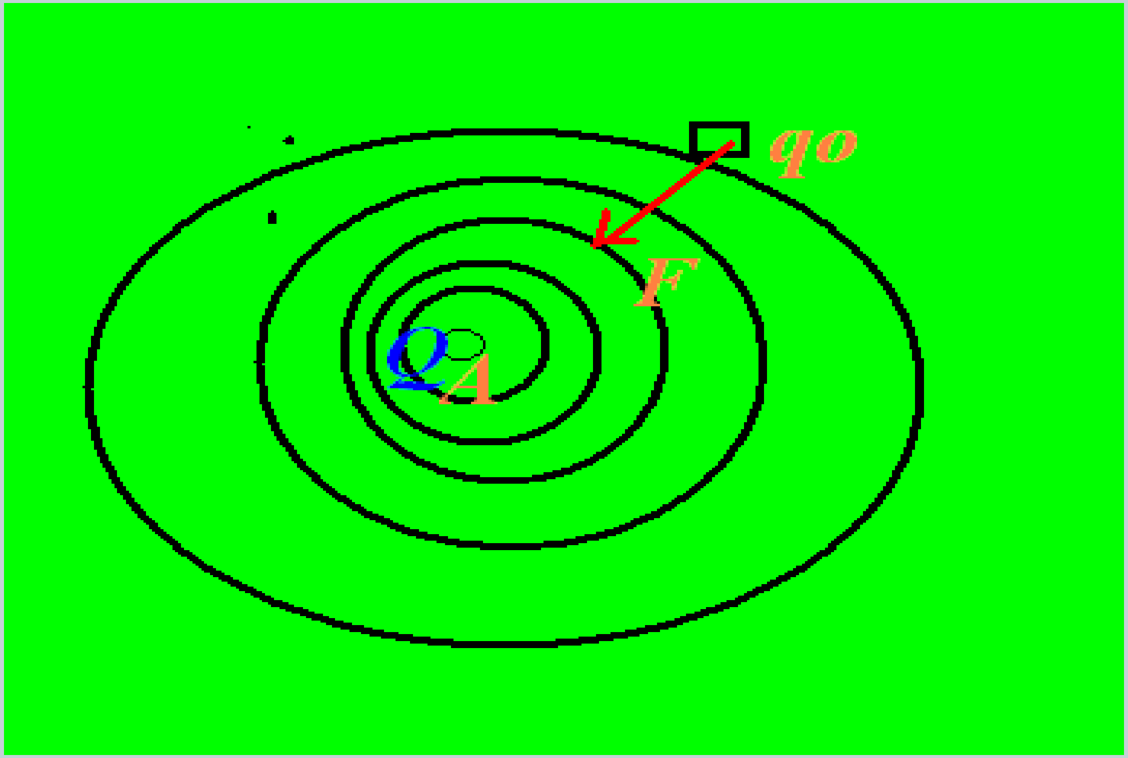
$$w = -QE.\int dL$$

- ---here dL called differential length
- *<u>Note:</u>
- dL= dx i +dy j + dz k-- for cartesian coordinates
- $dL = d_{\rho}\hat{a}_{\rho} + \rho d_{\phi}\hat{a}_{\phi} + d_{z}\hat{a}_{z}$--- for cylindrical
- $dL = d_{r}\hat{a}_{r} + rd_{\theta}\hat{a}_{\theta} + r\sin\theta d_{\phi}\hat{a}_{\phi}$ -- for spherical

➔ Potential & Potential Difference:

- Potential at any point P can be defined as..

"Work done to bring the unit charge from infinity to point P"

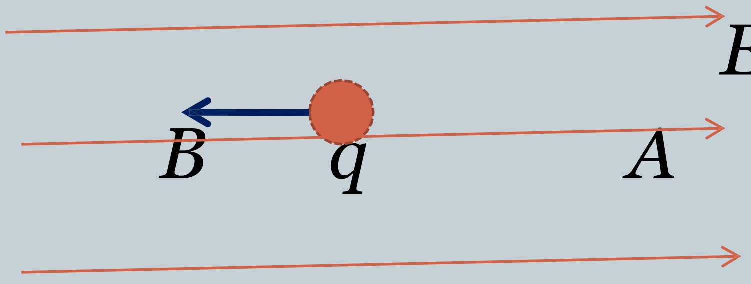


➔Potential Difference:

- *"Work done by external source in moving a positive unit test charge from one point to another point (in an electric field)"*

➔*Work done:*

- $$W_{AB} = -q\int_{A}^{B} E.dl$$



➔*Potential difference:*

$$V_{AB} = -\int_{A}^{B} E.dl$$

- → Or We Can say

$V_{AB} = V_A - V_B$: Where V_A – Potential at Point A

V_B – Potential at point B

: UNIT -→ –Joule/Coulomb‖ or Volt.

- Potential difference for

1: In electric field due to line charge distribution.

2: In electric field due to point charge at radial points.

➔ Equi potential Surface

- $V_{a+dl} = V_a + (dV/dL)dl$

  Let C1 constant rate of change in potential per unit distance.,

  Now, if $V_{a+dl} = V_a$

  ➔ $dV/dL = 0$ ➔ $V =$ Constant

➔ Hence Equipotntial surface can be defined as:

"Surface composed of all those points having the same value of potential. No work is involve in moving a unit test charge"

➔Potential Gradient

- ➔ As we Know the relation

$$V = -\int E.dl$$

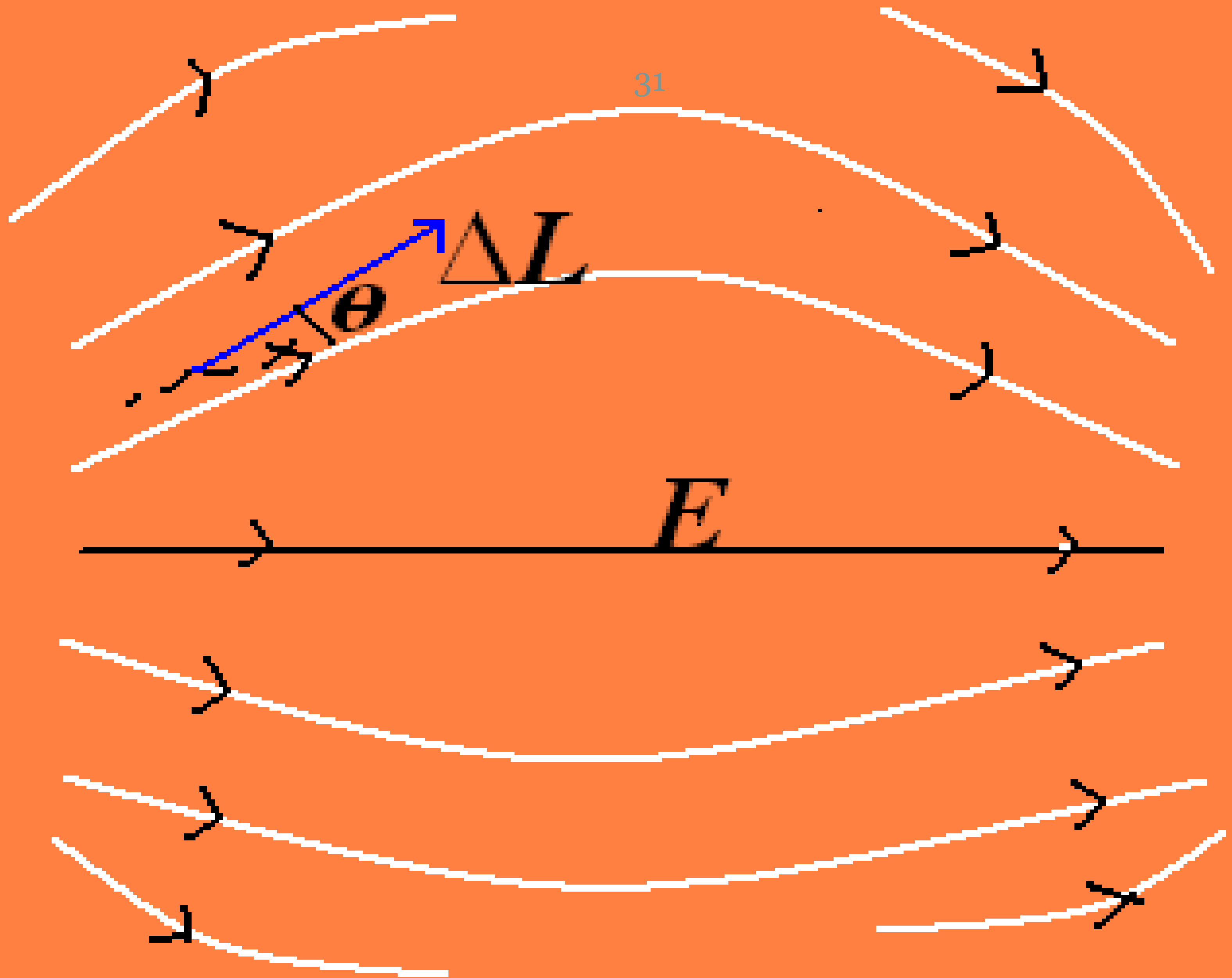
- But we can make this relation much easy using in reverse order:i.e

$$\Delta V = -E.\Delta L$$ ➔for short length

- Now, we suppose a region in which a vector is making angle θ with field direction:$\Delta L = \Delta L\, a_L$

So we will get the relation:

$$\Delta V = -E\Delta L Cos\theta$$



ΔL
θ
E

- ➔ Now we can convert this relationship in derivative by applying limits:

$$\frac{dV}{dL} = -E\cos\theta$$

➔ if we want to maximize this relationship the we must move in opposite to electric field($\cos\theta=-1$)

$$\left.\frac{dV}{dL}\right|_{max} = E$$

Here we observed two characteristics of relationship between E&V:

1: The magnitude of the electric field intensity is given by the maximum value of the rate of change of potential with distance.

2: the maximum value is obtained when the direction of distance increment is opposite to E.

Or we can say

"The direction of E is opposite to the direction in which potential is increasing the most rapidly."

- Now , if we consider *"Equipotential surface"* then we will get the relation:

$$\Delta V = -E.\Delta L = 0$$

Here we know that neither E nor ΔL can be zero. So E must be perpendicular direction of incremental vector ΔL.

So electric field intensity can be expressed se…

$$E - \left.\frac{dV}{dL}\right|_{max} \hat{a}_N$$

- ➔ so E can be expressed as *maximum rate of change in voltage with distance* and direction is *normal* to Equipotential surface.
- So, here we conclude *"(dV/dl)max occure when ΔL is in the direction of a_N"*
- ➔

$$\left.\frac{dV}{dL}\right|_{max} = \frac{dV}{dN}$$

$$\Longrightarrow E = -\frac{dV}{dN}\hat{a}_N$$

- ➔ The operation on V by which E is obtained is known as gradient.

E=-grad V

- ➔ ***Proof:***

Let V is an unique function of x,y,z and we take its complete diffrential.

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

- But we have also

$$dV = -E.dL = -E_x dx - E_y dy - E_z dz$$

- ➔ Comparing the both equations:

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

- ➔then result may be combined as:

$$E = -\left(\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right)$$

- ➔ $E = -gradV$
- ➔ $gradV = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z$

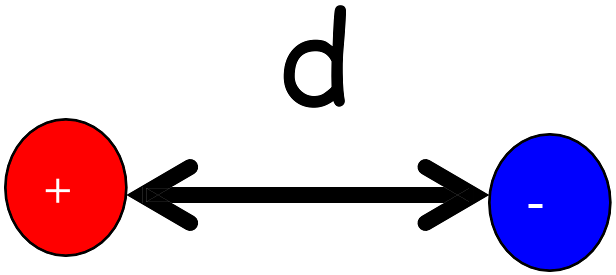
Gradient of V in other coordinate system:

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_{\phi} + \frac{\partial V}{\partial z}\hat{a}_{z}$$ ➔Cylindrical

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_{\phi}$$ ➔Spherical

Electric Dipole

- Electric dipole consists of a pair of point charges with equal size but opposite sign separated by a distance d.



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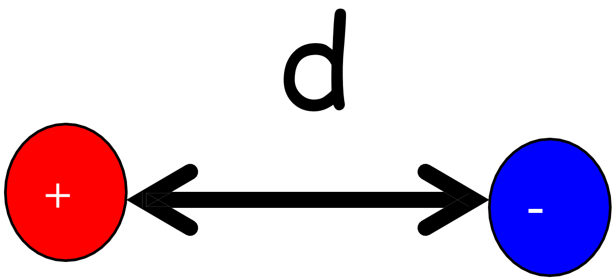

$$p = q \times d$$

Electric Dipole

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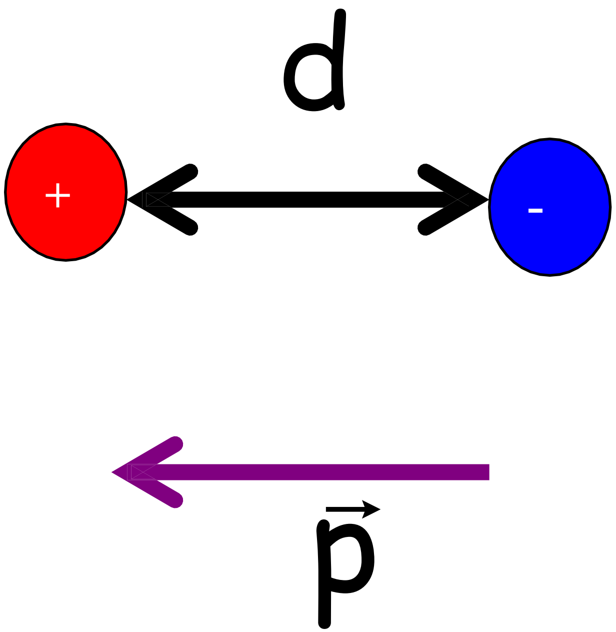
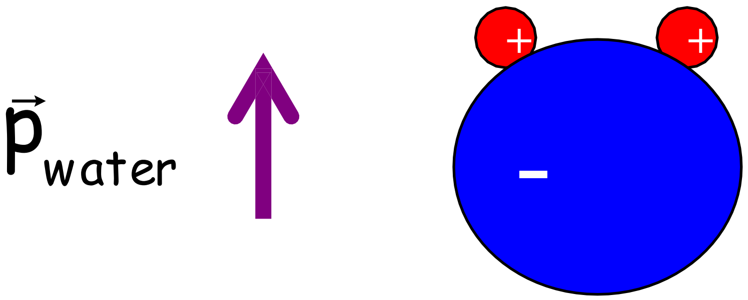
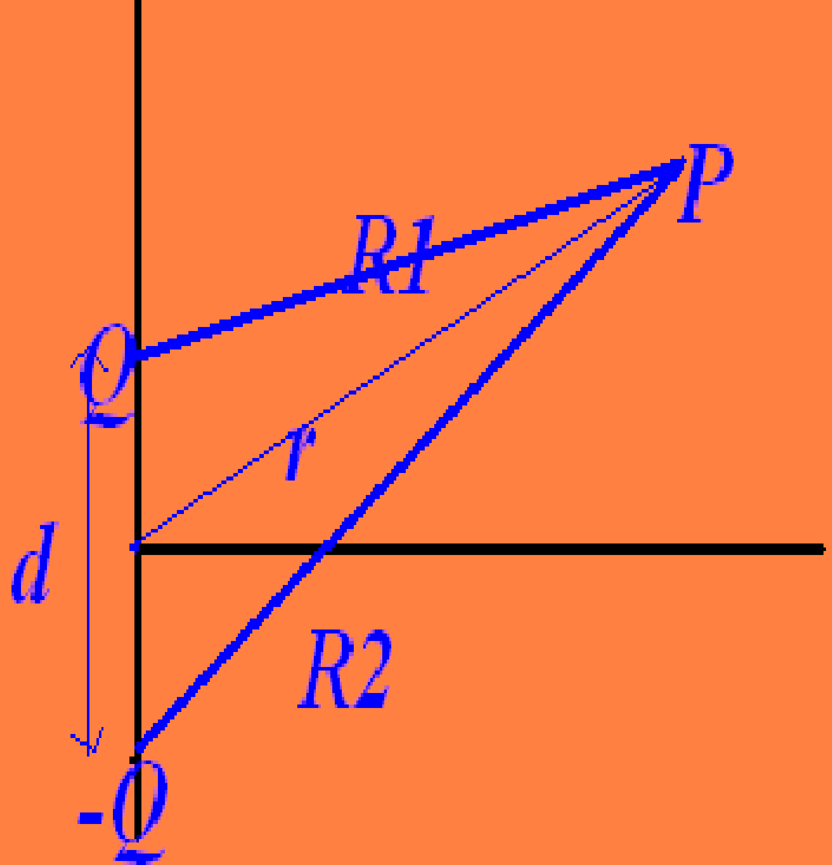


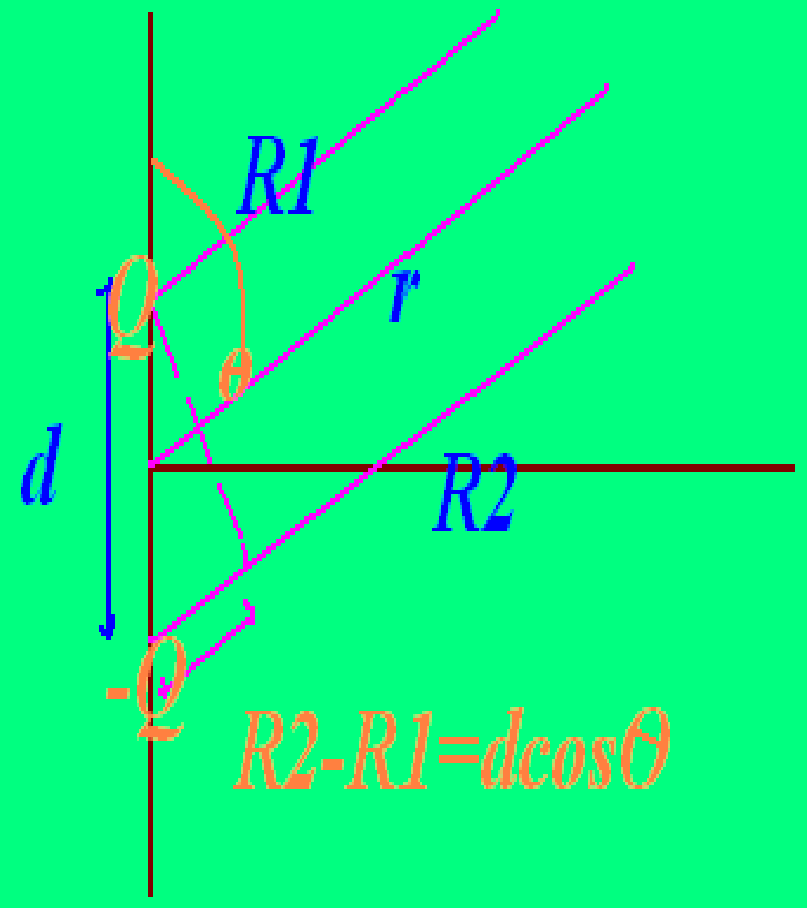
Electric Dipole

- Water molecules are electric dipoles



➔potential due to electric dipole





- ➔Potential at point P

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{R_2 - R_1}{R_1 R_2}$$

- R2-R1 may be approximated very easily if R1 and R2 are assumed to be parallel,
- R2-R1 = d cos θ
- ➔ The final result is then:

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

➔Now Electric field:

- By using gradient relationship for spherical coordinates:

$$E = -\nabla V = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi \right)$$

- So , we will get….

$$E = -\left(-\frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\hat{a}_r - \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^3}\hat{a}_\theta \right)$$

$$\Rightarrow E = \frac{Qd}{4\pi\varepsilon_0 r^3}\left[2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta \right]$$